SIMUL1ANE OUS RE1RIEVAI OF SURFACE AND RAIN BACKSCAT1ERING PARAME'1 ERS F ROM SPACEBORNE RADAR ME ASUREME N-IS

[astwood Im and Ziad S. Haddad

Jet Propulsion Laboratory, California Institute of 1 ethnology
Pasadena, California

1. IN-I RODUC1 ION

1 he precipitation radar planned for the '1 ropical Rainfall Measuring Mission ("I RMM) will be the first of its kind to measure vertical rainfall distributions from space. '1 he 1 RMM radar will scan ±l 20° across the nadir track. '1 he range-gated backscattering powers over the entire scan swath will be measured, classified (rain versus no- rain), averaged, and processed to derive the rainfall rates, With this observation scheme, there are two major reasons why it is important to know the rain-perturbed backscattering coefficient of the surface background ($\hat{\sigma}_0$). 1 irst, as the radar scans away from nadir, the return signals within the same pulse volume will include signals backscattered from both the rain and the surface background. By knowing $\hat{\sigma}_0$, the surface return can be removed and the rain rate near the surface can then be deduced. With the conventional approach, $\hat{\sigma}_0$ would be approximated by the rain-free coefficient σ_0 obtained either in the vicinity of the rainy area or from prior observations of the same area during dry periods, however, the error associated with such an approximation may significantly degrade the accuracy of the inferred rainfall intensities. It is therefore desirable to determine $\hat{\sigma}_0$ directly from the radar measurements acquired in the rainy area.

In this paper, we discuss a new algorithm for estimating $\hat{\sigma}_0$ as well as the reflectivity and attenuation coefficients in the rain above. 1 his algorithm is intended for use with single-frequency range–gated radar echo measurements such as those acquired by the 1RMM radar. Based on the expected 'I RMM radar performance characteristics, speckle is likely to be a major source of "noise" in the radar backscatter measurements, and we derive a maximum- likelihood estimator to minimize the speckle--induced errors in the retrieval of the surface backscattering coefficient. In addition to stochastic sources of error, the fact that only one frequency is available causes *deterministic* ambiguities to be present in the radar returns. We account for the stochastic and deterministic ambiguities in the retrieval of the rain characteristics by using an optimal non-linear filtering approach.

"1 he details of the problem and the algorithms we derive are described in the following sections. 1 his research was carried out at the Jet Propulsion 1 aboratory, California Institute of '1 ethnology, under contract with the National Aeronautics and Space Administration,

2. 1 HE INVE RSION PROBLE M

We consider the situation pictured in figure 1. Specifically, calling the radar beam incidence angle ϕ and the range resolution dr, we assume that we receive J pieces of rain–only data, namely 'the echo powers q_j from ranges IL's - j dr,

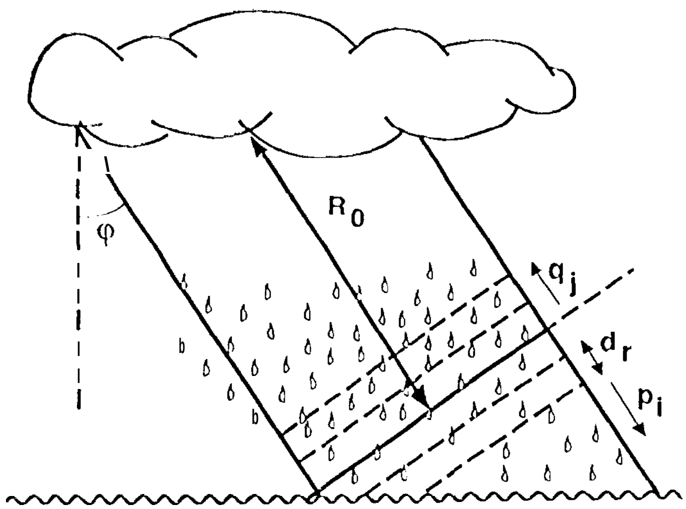

Figure 1: off- nadir viewing geometry

$$q_j = \text{echo from range } R_0 - j\,dr,\ 1 \le j \le J$$
$$\simeq (\eta A e^{-\alpha_j k} + \Sigma^2) \cdot U_j \qquad (1)$$

where

η = rain reflectivity coefficient

k: rain attenuation coefficient

A: (known gain) (calibration constant)

α_j = $0.2 \log(10)\,(R_0 - j\,dr)$

Σ^2 , thermal noise variance

U_j , speckle variance at range $R_0 - j\,dr$

followed by N pieces of surface-cluttered data, namely the echo powers p_i from ranges $R_0 + i\,dr$,

$$p_i \;:\; \text{echo from range } R_0 + i\,dr,\; 0 \leq i \leq N-1$$
$$\simeq \left(\eta \hat{A}_i e^{-\hat{\alpha}_i k} + \hat{\sigma}_0 B_i e^{-\beta k} + \Sigma^2\right) V_i \qquad (2)$$

where, this time,

- $\hat{\sigma}_0$: rain-modified surface backscattering coeff
- $\hat{A}_i, B_i$: beam-filling-dependent gains
- $\hat{\alpha}_i$: $0.2\log(10) \cdot (R_0 + i\,dr)$
- β : $0.2\log(10) \cdot R_0$
- V_i : speckle variance from range $R_0 + i\,dr$

Since we have $J+N$ equations in three unknowns, one would expect the problem of determining $\eta, \hat{\sigma}_0$ and k to be easy to solve. However, the data are contaminated by speckle noise, whose variance can be of the same magnitude as the parameters we need to estimate. The best approach is to try to make a statistically optimal estimate.

Specifically, assuming that each U_j is the arithmetic average of the squared magnitude of M independent complex standard normal random variables, where M is the number of radar pulses transmitted along one fixed scan angle ($M \simeq 60$ for 1 RMM), the probability density function f for each U_j is

$$f(u) = M\frac{(Mu)^M}{(M-1)!}\,e^{-Mu} \qquad (3)$$

It follows from the equation for f that the maximum likelihood estimator for η, k given the data q_j is obtained by looking for the values $\hat{\eta}, \hat{k}$ of η, k which minimize the quantity

$$\frac{1}{J}\sum_{j=1}^{J} \frac{q_j}{F_j(\eta,k)} - \log\frac{q_j}{F_j(\eta,k)} \qquad (4)$$

where $F_j(\eta,k) = \eta A e^{-\alpha_j k} + \Sigma^2$. Once the maximum likelihood estimates $\hat{\eta}, \hat{k}$ are determined, one must similarly look for the value of $\hat{\sigma}_0$ which minimizes the likelihood function

$$\frac{1}{N}\sum_{j=0}^{N-1} \frac{p_i}{G_i(\hat{\eta},\hat{\sigma}_0,\hat{k})} - \log\frac{p_i}{G_i(\hat{\eta},\hat{\sigma}_0,\hat{k})} \qquad (5)$$

where $G_i(\hat{\eta},\hat{\sigma}_0,k) = \hat{\eta}\hat{A}_i e^{-\hat{\alpha}_i k} + \hat{\sigma}_0 B_i e^{-\beta k} + \Sigma^2$. This would in principle determine the optimal estimate for $\hat{\sigma}_0$. Yet, in order to derive this first-cut algorithm, we have made one implicit assumption that is not realistic.

Indeed, note that we have not specified the values of J or N. In fact, N is completely determined by the geometry. However, J can a priori be arbitrary. In reality, we cannot allow J to be too large, for we would then be assuming that η and k are constant over a long slant distance $J\,dr$, a generally unjustifiable hypothesis. But if J is small, we would be left with too little data to beat down the speckle noise. We must therefore look for a way to estimate η and k in the general case where they are not a priori assumed to be constant.

We are thus naturally led to assume that k is in fact a function of range, $k = k(r)$, which we must determine using the measured echo power data q. So far, we had been writing q as a discrete variable. For consistency, we now represent the rain echo power data as a function of continuous range $q(r)$. Rather than introduce yet another unknown function $\eta(r)$, we assume a power-law k–η relation $\eta = \delta k^\gamma$, and set out to estimate the unknowns $k(r), \delta, \gamma$ given the data $q(r)$.

Since k is now an unknown *function*, it would be quite unwieldy to discretize it and attempt a mamximum likelihood approach. On the other hand, we can consider it a stochastic process, with the range variable r playing the role of time, then try to use optimal stochastic filtering techniques. Indeed, if we represent the relationship between the data q and the unknowns k, δ, and γ by the equation

$$q(r) = \left(A\,\delta\,k(r)^\gamma\,10^{-0.2c(r)} + \Sigma^2\right) \cdot U(r) \qquad (6)$$

where $c(r) = \int_0^r k(t)\,dt$ is the cumulative attenuation, and if we make some simple assumptions about the dynamics of k, i.e. about its behavior as a function r, we should be able to derive the differential equation governing the evolution with r of the probability density function $\mathcal{P}_r(k,\delta,\gamma)$ of k,δ,γ at range r, conditioned on the data $\{q(t), t \leq r\}$. Furthermore, if our model for the dynamics is indeed simple, we might be able to solve the differential equation explicitly, thus obtaining an algorithm for estimating $k(r), \delta$, and γ. We would then be able to use $\hat{k} = k(R_0)$ and $\hat{\eta} = \delta k(R_0)^\gamma$ as our estimates for the near-surface reflectivity and attenuation coefficient in order to find the maximum-likelihood estimate for $\hat{\sigma}_0$ as described earlier.

Thus, as soon as we settle on a simple model for the dynamics of k, we should be able to write down the corresponding optimal algorithm to